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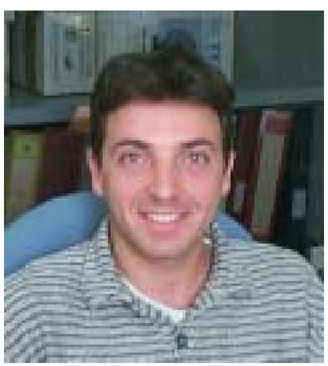
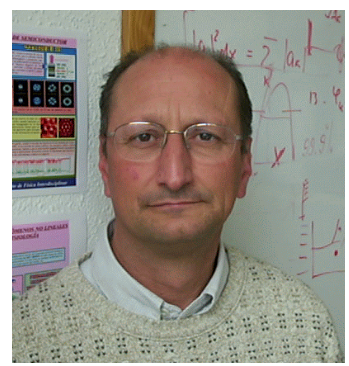
Localized Structures in the Parametrically Driven CGLE


Damià Gomila


Pere Colet


Maxi San Miguel


Gian-Luca Oppo

Phys. Rev. Lett. 87, 194101 (2001)

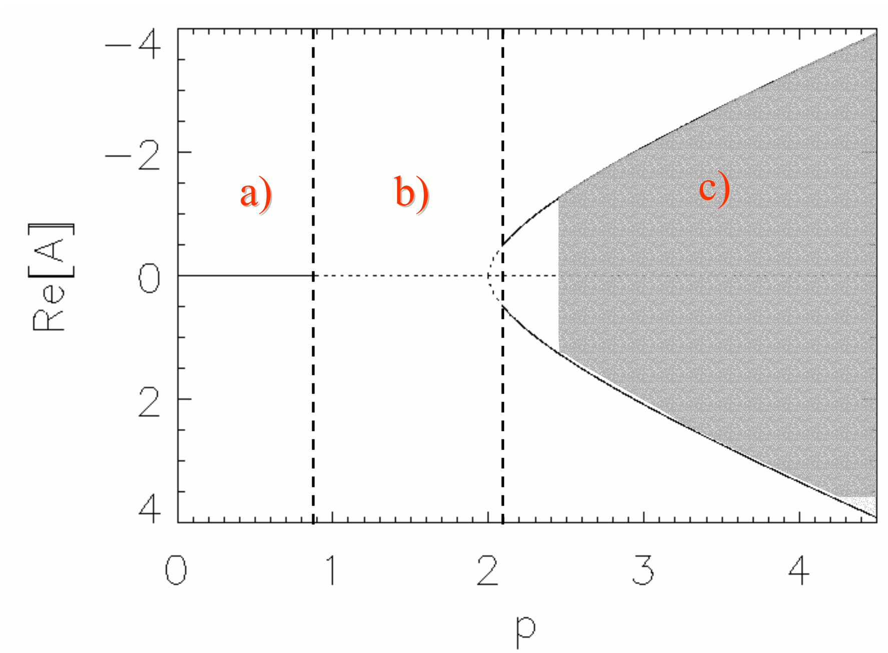
Parametrically Driven CGLE

$$\partial_t A = (\mu + i\upsilon)A + (1 + i\alpha)\nabla^2 A - (1 + i\beta)|A|^2 A + pA^*$$

$\alpha \sim \nu \sim p >> \mu, \beta$ → Pattern forming transition

P. Coullet and K. Emilsson, Physica D, 61, 119 (1992)

Homogeneous solutions:



a) Zero solution

b) Stripe and hexagonal Patterns

c) Bistability between homogeneous solutions (frequency locked solutions).

Stable Ising walls.

$\alpha = \nu = 2, \mu = \beta = 0$

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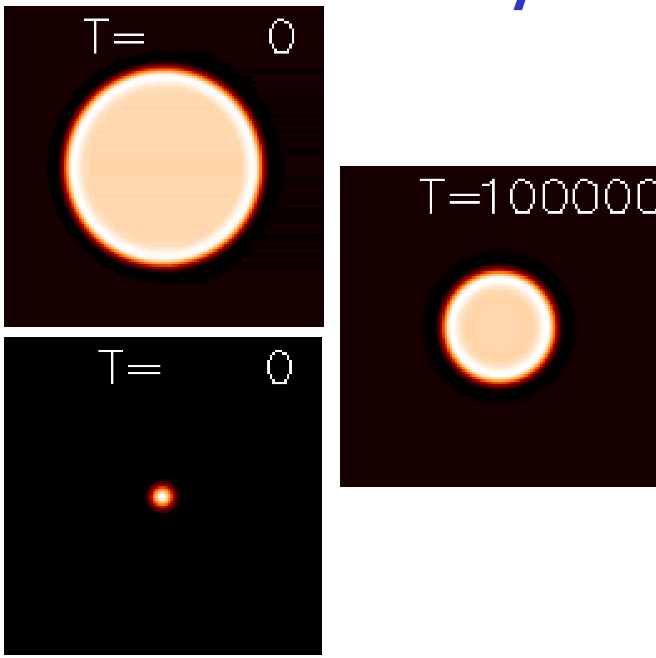
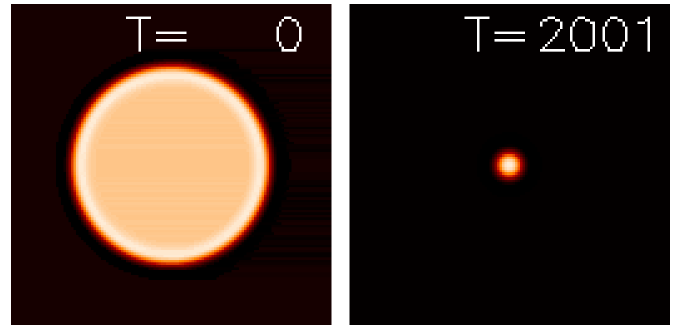
Dynamics of a Circular Domain

Coarsening

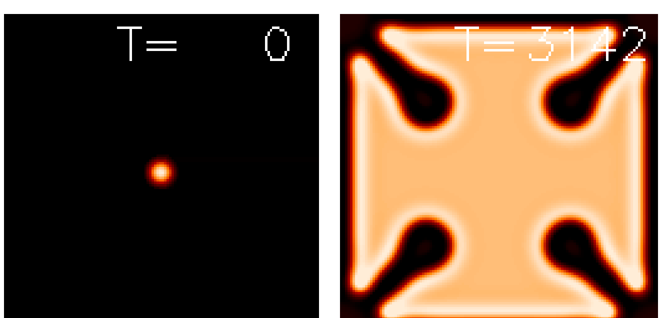
Stable droplet

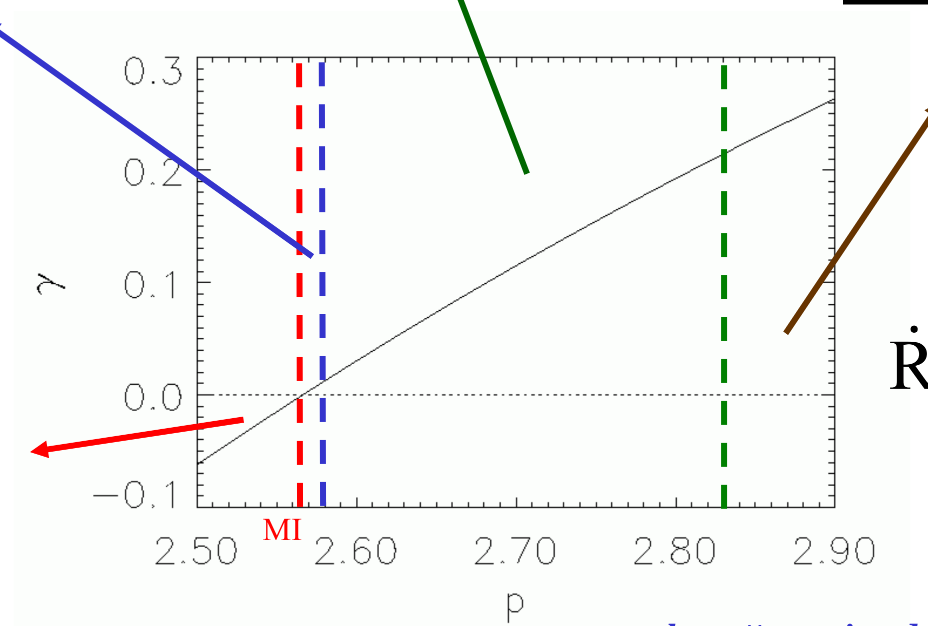


Localized Structure



Exploding droplet





$$\dot{R} = -\gamma \frac{1}{R}$$

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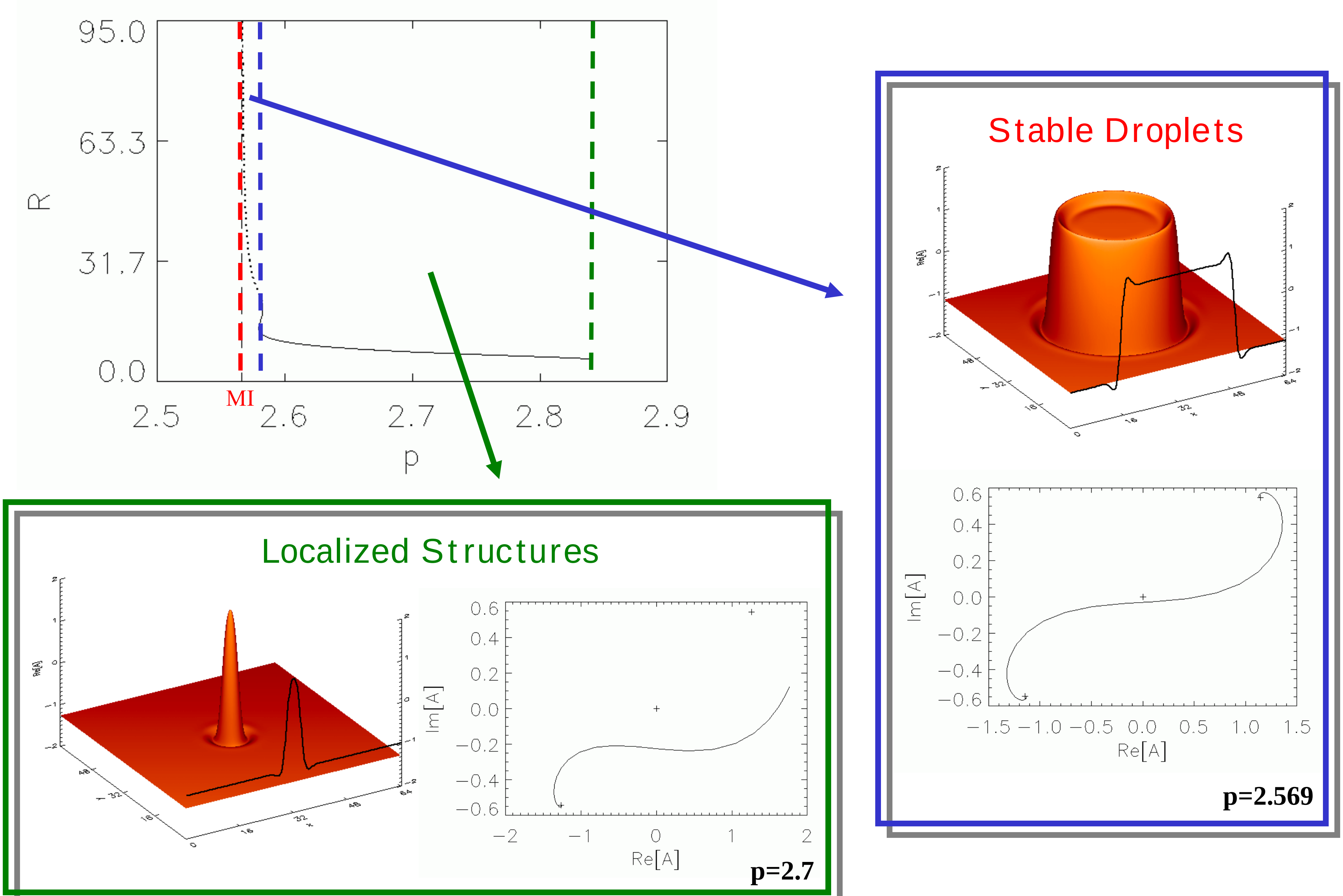
Localized Structures vs Stable Droplets



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Curvature Driven Dynamics

$$\partial_t \vec{A}(\vec{x}) = D\nabla^2 \vec{A}(\vec{x}) + \vec{W}(\vec{A}(\vec{x}), p)$$

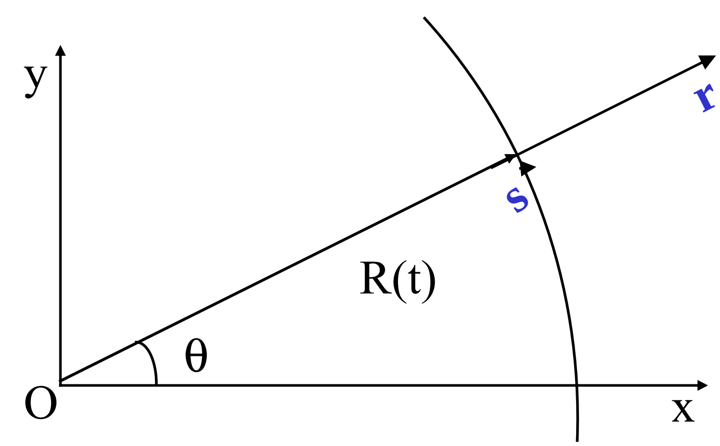
- $\vec{A}$ is a real vector field (For the PCGLE $\vec{A} = \begin{pmatrix} \mathrm{Re}[A] \\ \mathrm{Im}[A] \end{pmatrix}$ and $D = \begin{pmatrix} 1 & -\alpha \\ \alpha & 1 \end{pmatrix}$).
- $\vec{W}(\vec{A})$ is a local nonlinear function of the fields.
- There exists a discrete symmetry ***Z*** of $\vec{W}(\vec{A})$ such that there are two equivalent stable homogeneous solutions (For the PCGLE ***Z*: *A*** $\rightarrow$ ***-A***).
- Parameter region with stable d=1 Ising walls $\vec{A}_0(x, p)$.

Local Coordinates r, s

$$\partial_t \rightarrow -v\,\partial_r \qquad (v \equiv \dot{R})$$

$$\nabla^2 \rightarrow \partial_r^2 + \frac{\kappa}{1+r\kappa}\partial_r + \frac{\kappa^2}{(1+r\kappa)^2}\partial_\theta^2 \qquad (\kappa \equiv \frac{1}{R})$$

E. Meron, Phys. Rep. ***218*** *(1992)*



Curvature Driven Dynamics

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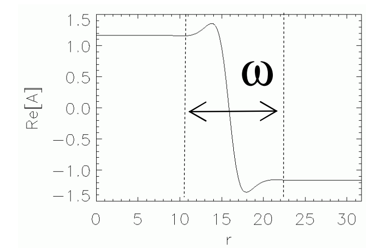
$$D\partial_r^2 \vec{A}(r) + (\mathrm{v} + \frac{\kappa}{1+r\kappa} D)\partial_r \vec{A}(r) + \vec{W}(\vec{A}(r),\mathrm{p}) = 0$$

Linearizing around the 1D profile: $\vec{A}(r) = \vec{A}_0 + \vec{A}_1$

$$M\vec{A}_1 = -(\mathrm{v} + \kappa D)\partial_r \vec{A}_0 \,, \quad M \equiv D\partial_r^2 + \left.\frac{\delta \vec{W}}{\delta \vec{A}}\right|_{\vec{A}_0}$$

Gently curved fronts
$\kappa\omega << 1$



$\vec{e}_0$ Goldstone Mode: $\vec{e}_0 \equiv \partial_r \vec{A}_0 \,, \; M\vec{e}_0 = 0 \,; \; M^+ \vec{a}_0 = 0$

Solvability condition: $\mathrm{v} = -\gamma(\mathrm{p})\kappa$

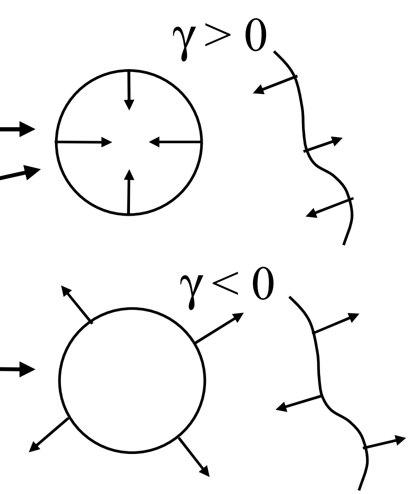
$$\gamma(\mathrm{p}) \equiv \frac{\int_{-\infty}^{\infty} \vec{a}_0^{\,+} \cdot D\vec{e}_0 \, dr}{\int_{-\infty}^{\infty} \vec{a}_0^{\,+} \cdot \vec{e}_0 \, dr}$$

Circular domain: $\dot{\mathrm{R}} = -\gamma \frac{1}{\mathrm{R}}$

- $D = dI \;\Rightarrow\; \gamma = \mathrm{d} > 0, \; \mathrm{v} = -d\kappa$ — $\gamma > 0$ Droplet shrinking

  $\vec{A}_1 \propto \vec{e}_0$, no deformation
- $D = dI + C \Rightarrow \gamma = \gamma(\mathrm{p})$, $\mathrm{v} = -\gamma\kappa$
  - $\gamma > 0$ Droplet shrinking
  - $\gamma = 0$ Modulational instability
  - $\gamma < 0$ Droplet growth

  $\vec{A}_1 \not\propto \vec{e}_0$, deformation $\propto \kappa$, C



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Amplitude Equation for the curvature, $\gamma = 0$

Radial symmetry ⟹ $$D\partial_r^2 \vec{A}(r) + \left(-\frac{\dot{\kappa}}{\kappa^2} + \frac{\kappa}{1+r\kappa} D\right)\partial_r \vec{A}(r) + \vec{W}(\vec{A}(r), \mathrm{p}) = 0$$

$$\mathrm{p} = \mathrm{p}_c + \varepsilon \mathrm{p}_1 \,,\; \gamma(\mathrm{p}_c) = 0 \,,\; \varepsilon << 1 \,,$$

$$\vec{A} = \vec{A}_0 + \varepsilon^{1/2}\vec{A}_1 + \varepsilon \vec{A}_2 + \varepsilon^{3/2}\vec{A}_3 \,,$$

$$\kappa = \varepsilon^{1/2}\kappa_1 \,,\; \partial_t = \varepsilon^2 \partial_T$$

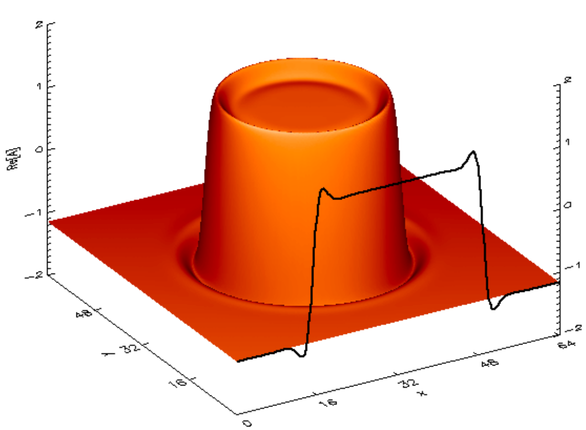
$$\Rightarrow \quad O(\varepsilon^{3/2}) \quad \frac{\partial_T \kappa_1}{\kappa_1^2} = c_1 \mathrm{p}_1 \kappa_1 + c_3 \kappa_1^3$$

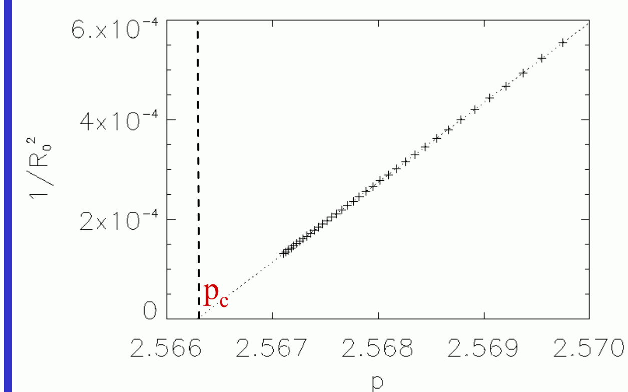
$c_1 > 0 \,, c_3 < 0$

$$\dot{\mathrm{R}} = -c_1(\mathrm{p} - \mathrm{p}_c)\frac{1}{\mathrm{R}} - c_3 \frac{1}{\mathrm{R}^3} \quad \Rightarrow$$

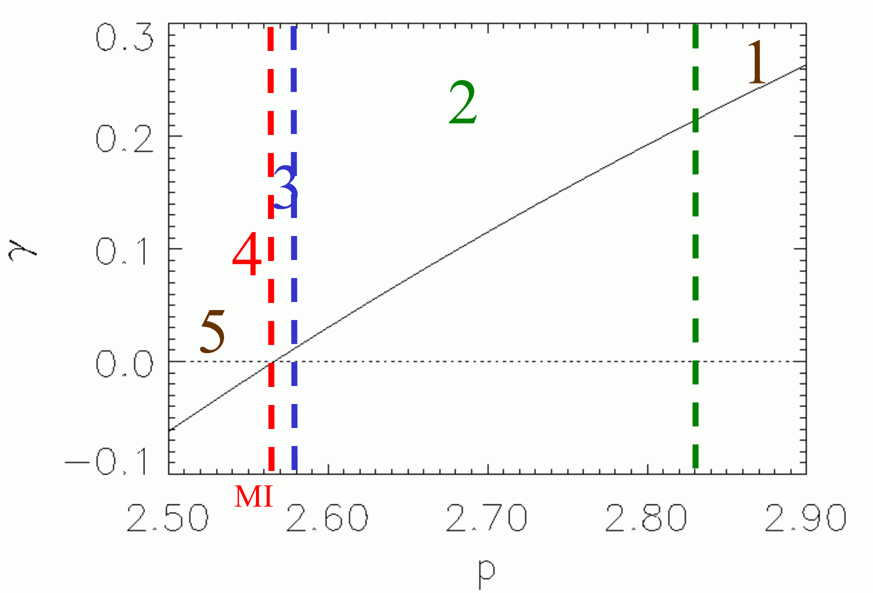
Stable Droplet

$$\mathrm{R}_0 = \frac{1}{\sqrt{\mathrm{p} - \mathrm{p}_c}} \sqrt{\frac{-c_3}{c_1}}$$





Growth Laws



1, 5 → $\dot{R} = -\gamma \frac{1}{R}$, $R \approx t^{1/2}$ **shrinking, growth**

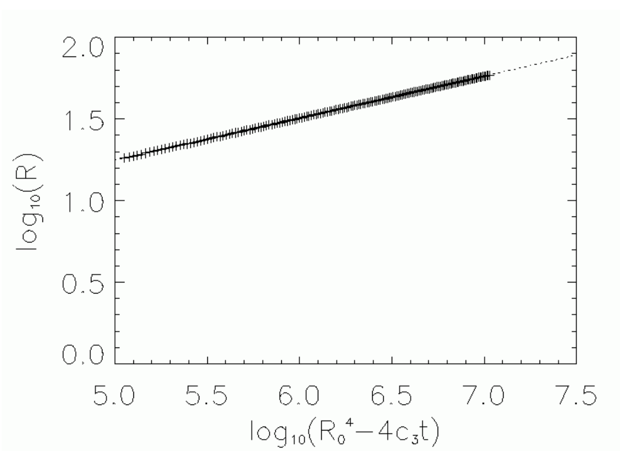
2 → $\dot{R} = -\gamma \frac{1}{R}$, $R \approx t^{-1/2}$ **until LS forms**

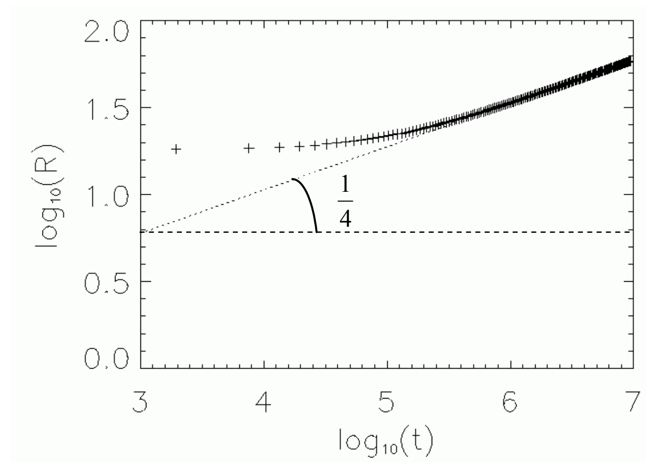
3 → $\dot{R} = -\gamma \left(\frac{1}{R} - \frac{R_0^2}{R^3} \right)$ **no power law**

4 → $\dot{R} = -c_3 \frac{1}{R^3}$,

$R \approx t^{1/4}$

modulational instability, $\gamma = 0$





Dynamics of Arbitrary Domains

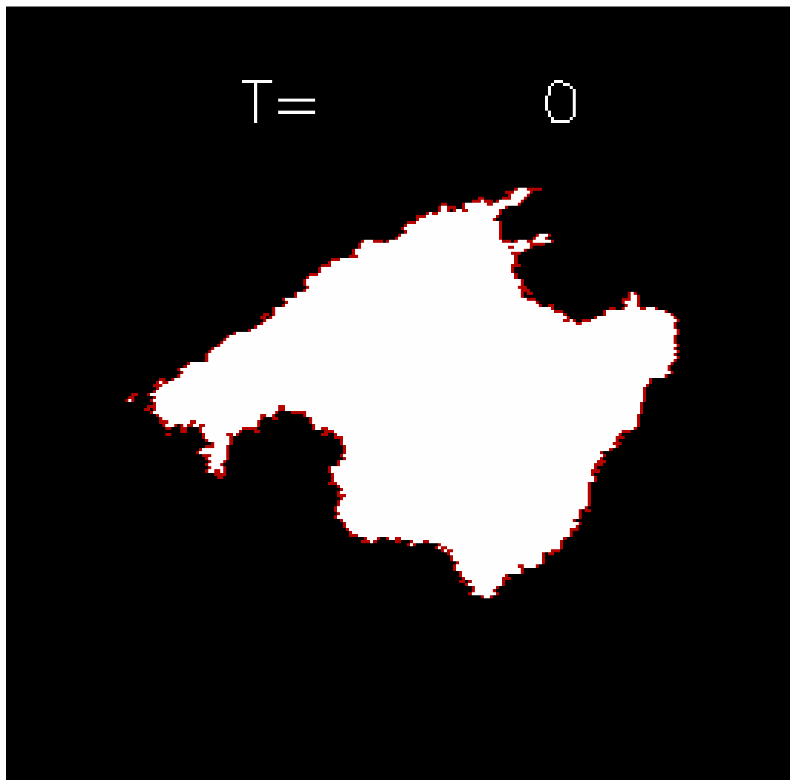
Normal front velocity: $v = -c_1(p - p_c)\kappa \; - c_2\kappa^2\partial_\theta^2\kappa \; - c_3\kappa^3$ $c_1 > 0, \quad c_2, c_3 < 0$

Shrinking term $O(\kappa)$

Reduction of curvature differences $O(\kappa^2)$

Exploding term $O(\kappa^3)$

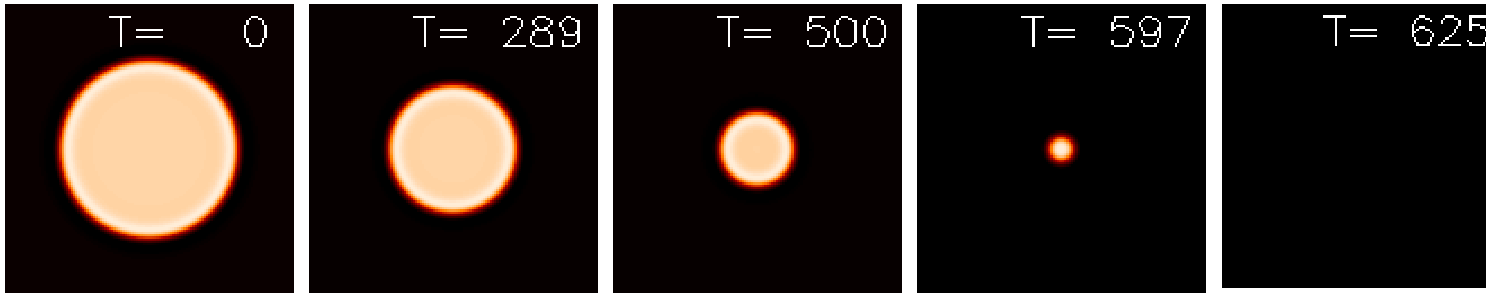
At the Modulational Instability $p = p_c$: $v = -c_2\kappa^2\partial_\theta\kappa \; - c_3\kappa^3$



Dynamics of a Circular Domain

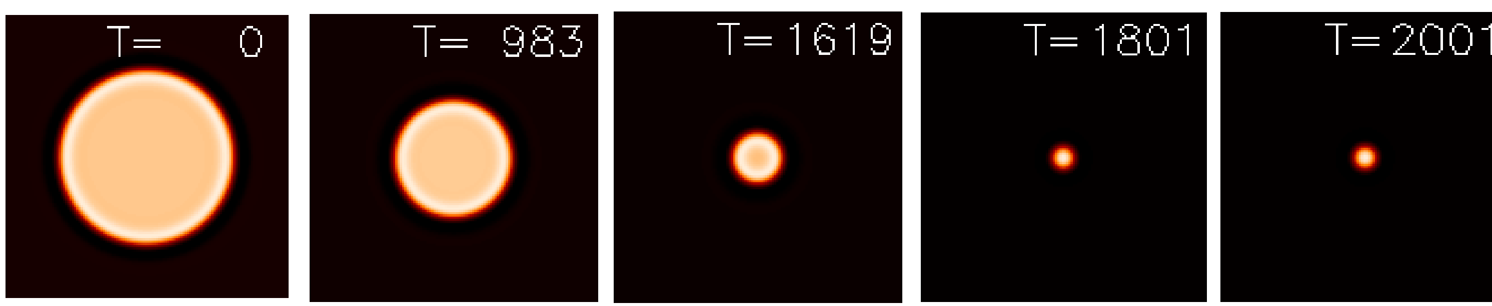
- Coarsening

  p = 3.0
  γ(p) = 0.3



- Localized Structure

  p = 2.7
  γ(p) = 0.1

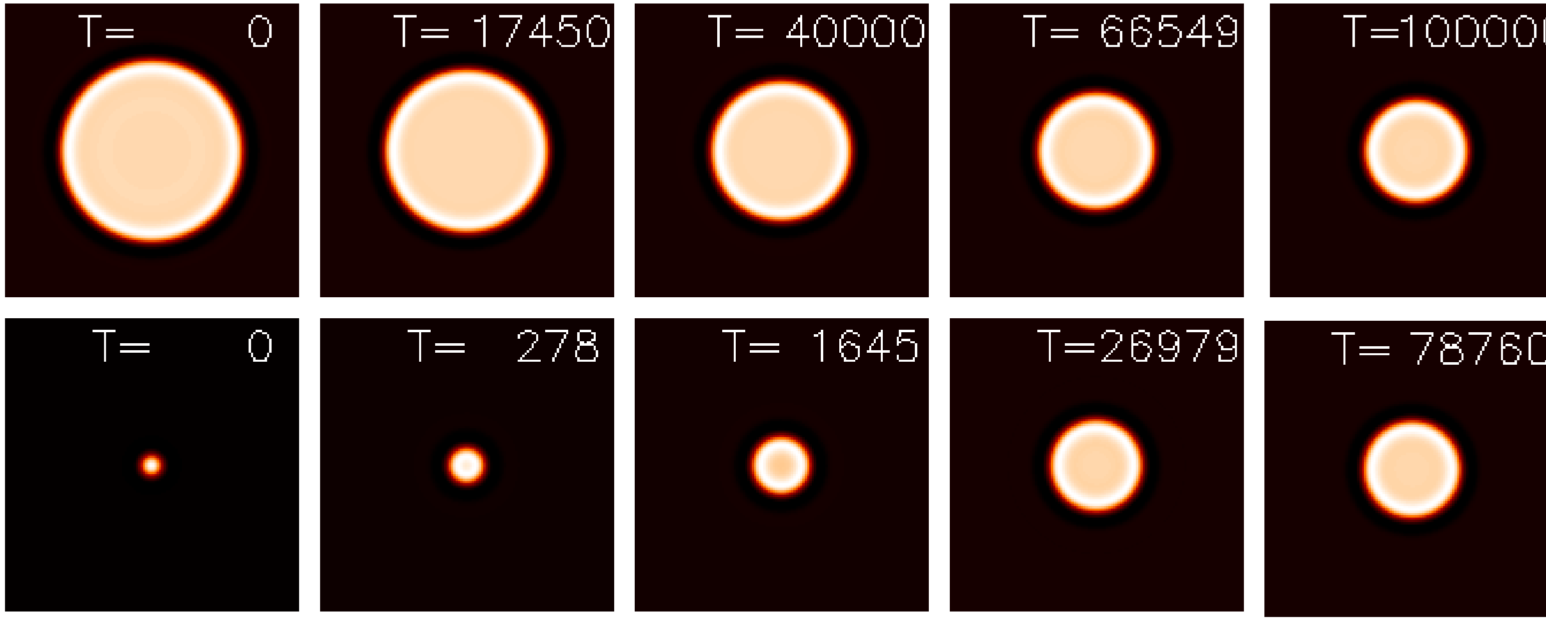


Dynamics of a Circular Domain

- Stable Droplet

p = 2.569

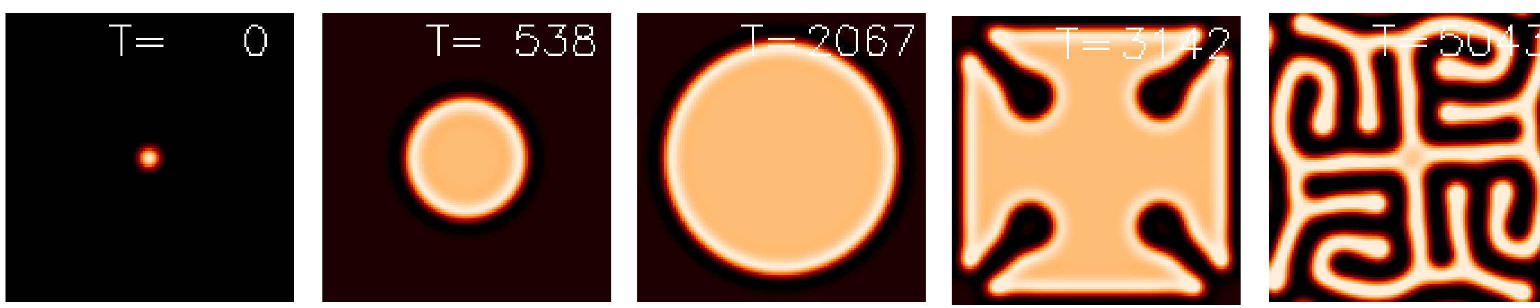
γ(p) = 0.01



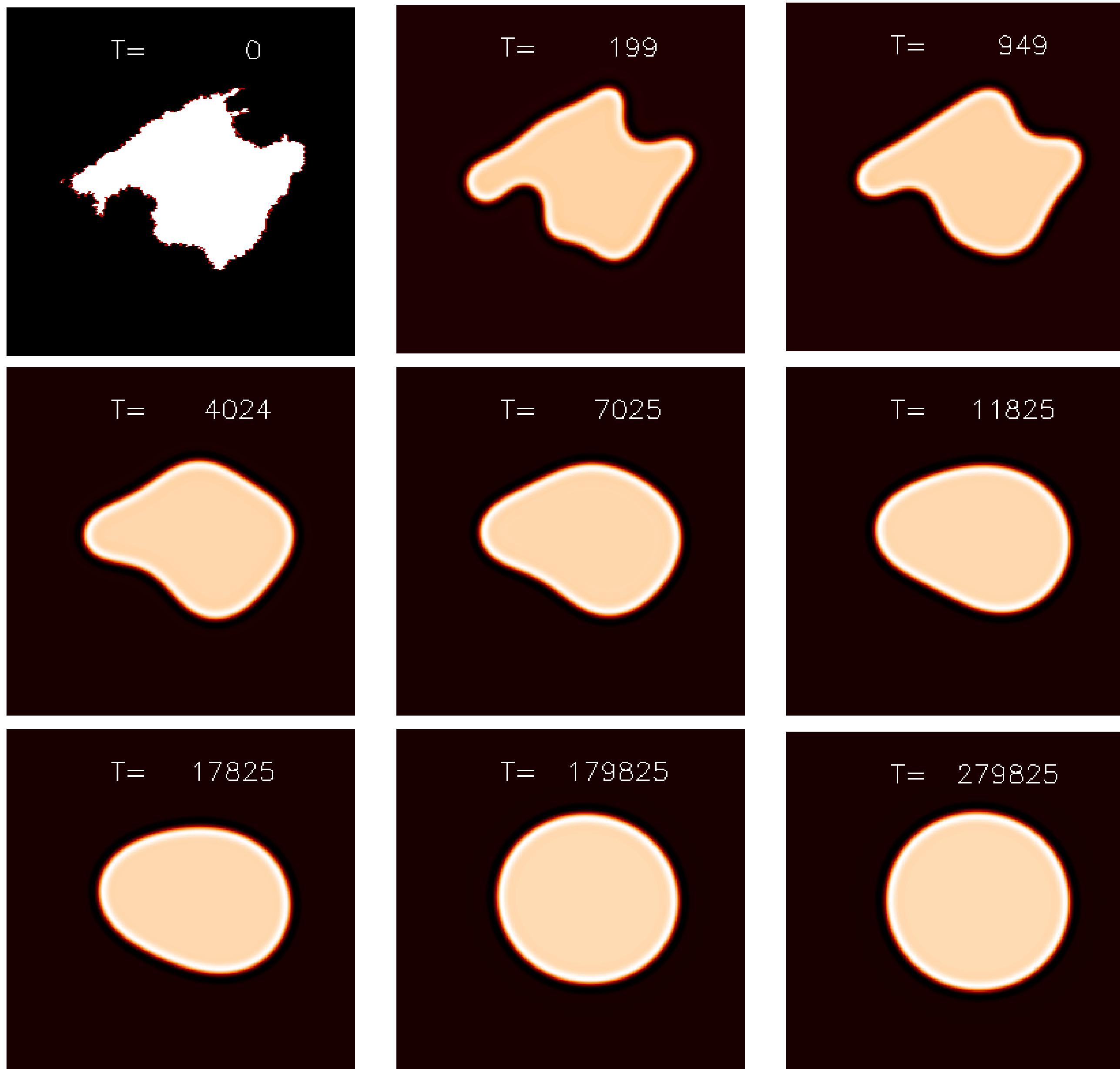
- Exploding Droplet

p = 2.4

γ(p) = –0.1



Dynamics of an Arbitrary Domain



$p = p_c$

$\gamma(p) = 0.0$