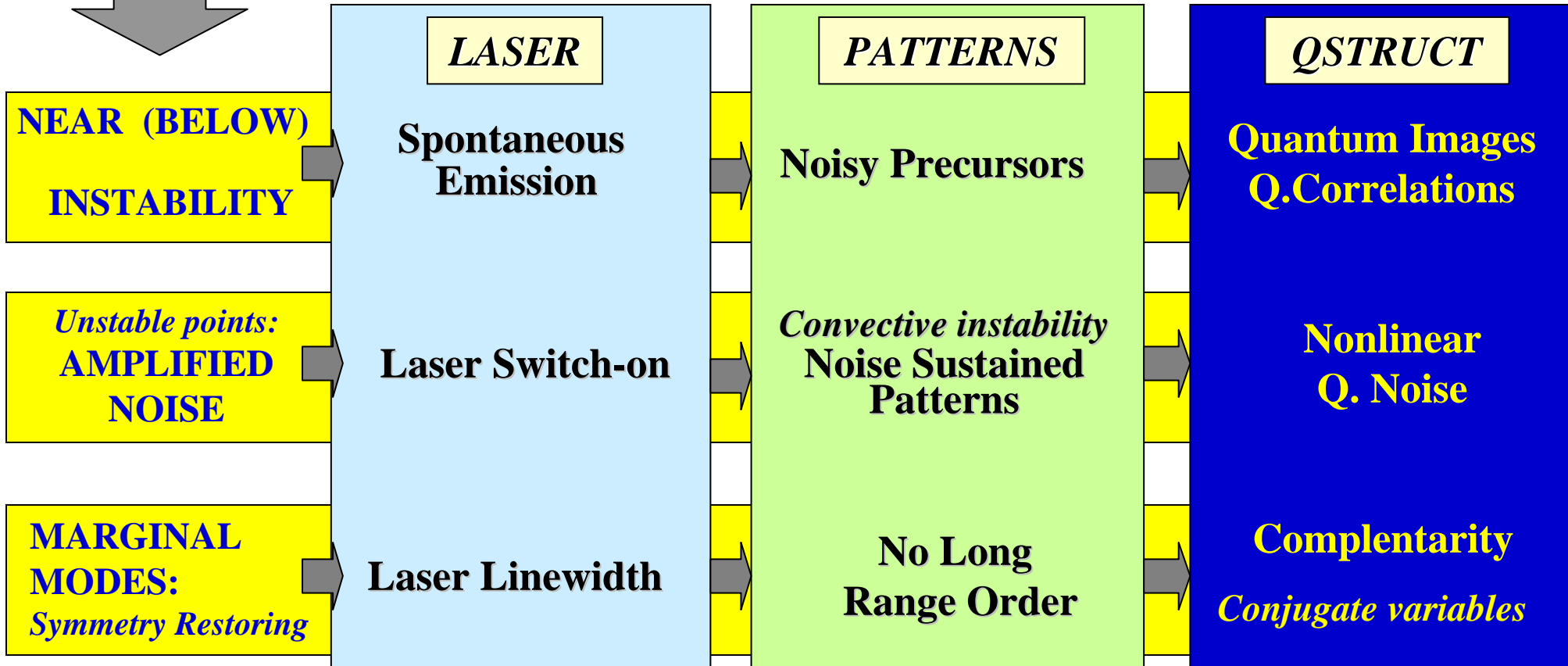
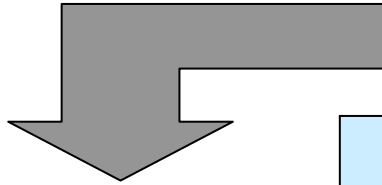


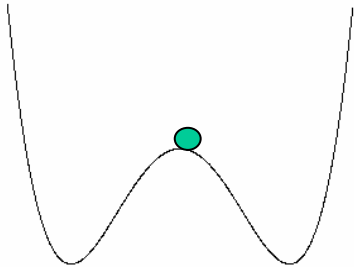
WHEN IS NOISE IMPORTANT?



NOISE INDUCED TRANSITIONS
STOCHASTIC RESONANCE

ENTANGLEMENT?

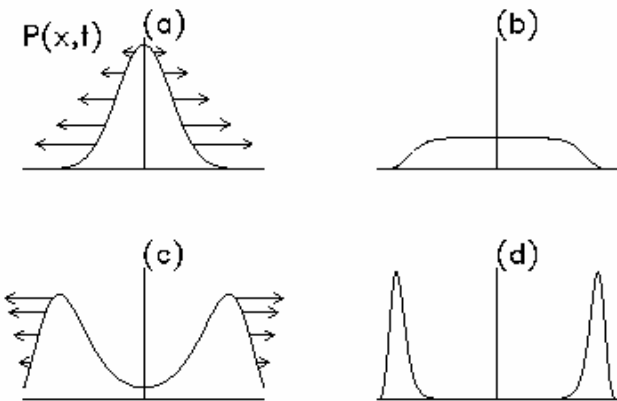
NOISE AMPLIFICATION: Decay of an unstable state



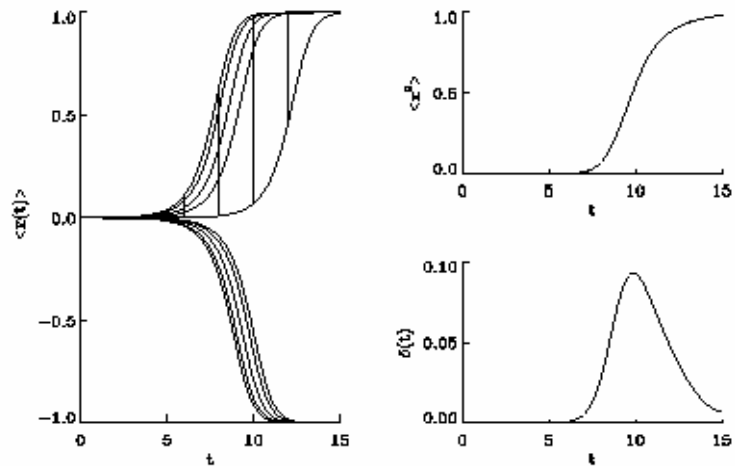
$$d_t x = ax - x^3 + \sqrt{\epsilon} \xi(t)$$

$$a > 0, \quad x(0) = 0$$

Probability distribution:

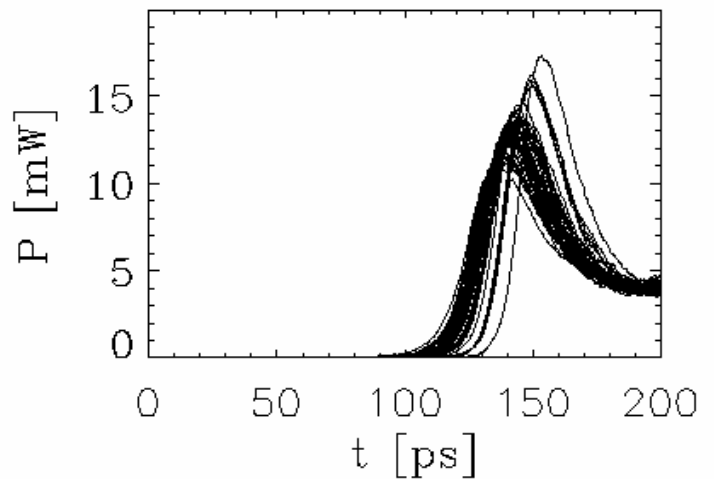


Trajectories:



LASER SWITCH-ON

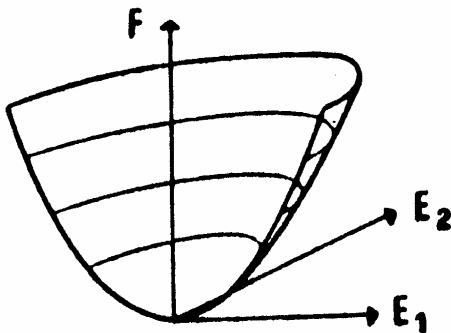
- Amplification of spontaneous emission noise
- Jitter



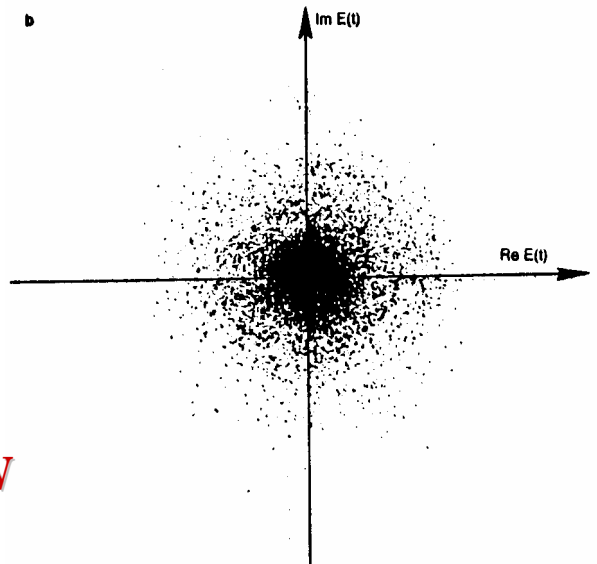
LASER FLUCTUATIONS

$$\partial_t E = aE - |E|^2 E + \sqrt{\varepsilon} \xi(t) = -\frac{\partial F}{\partial E^*} + \sqrt{\varepsilon} \xi(t), \quad E = \sqrt{I} e^{i\varphi}$$

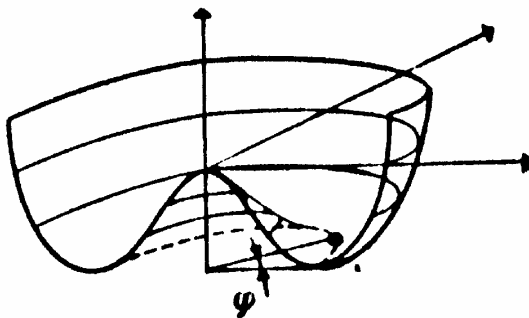
BELOW THRESHOLD



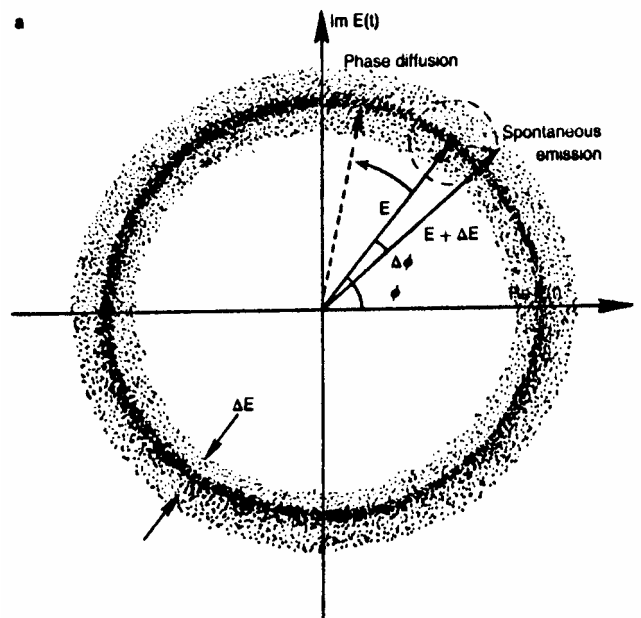
SPONTANEOUS EMISSION



ABOVE THRESHOLD



PHASE DIFFUSION

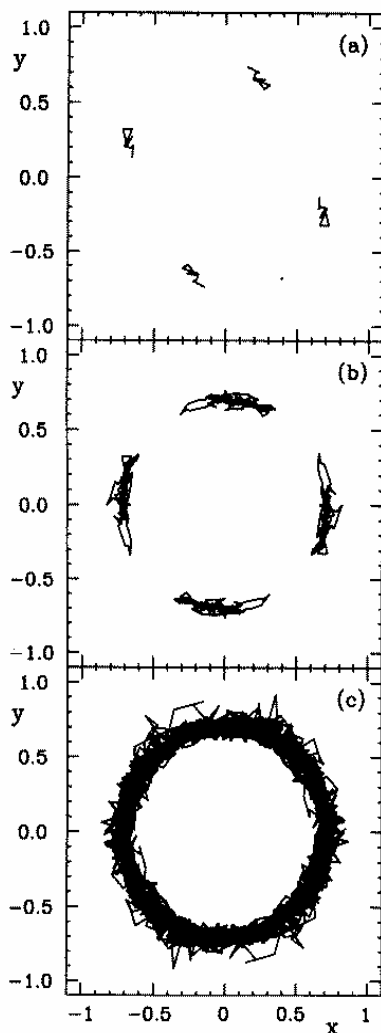


FLUCTUATIONS IN TRANSVERSE LASER PATTERNS

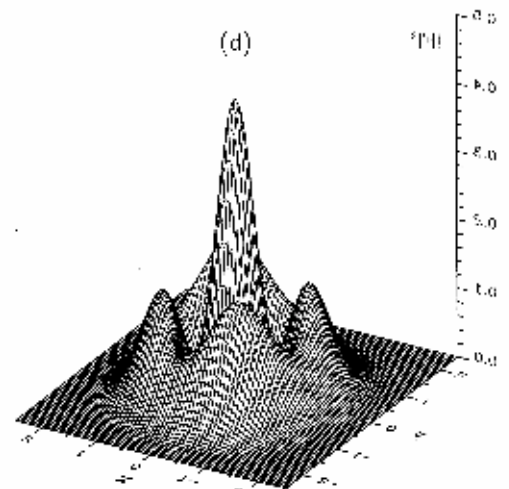
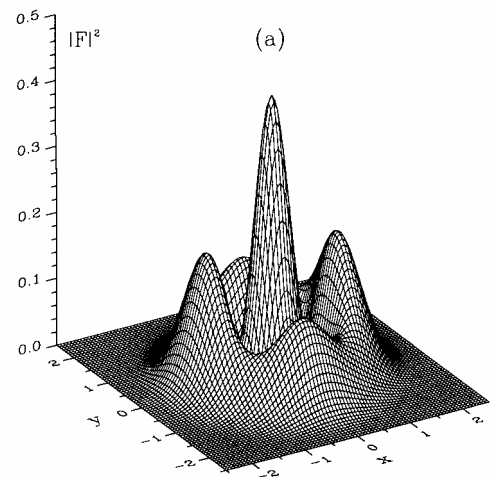
$$E(\rho, \varphi) = \sum_{i=1}^3 f_i(t) A_i(\rho, \varphi)$$

$$d_t f_i = -f_i + 2C \left(M_i f_i - \sum_{i=1}^3 A_{ijkl} f_j f_k f_l^* \right) + \xi_i(t)$$

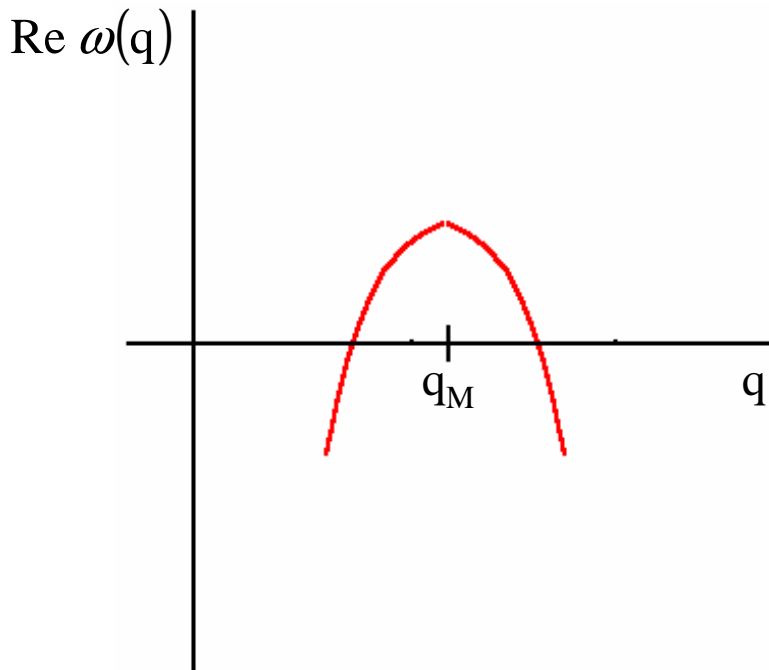
*Marginal phase associated with pattern orientation diffuses.
Broken rotational symmetry restored by noise*



time



PHASE DIFFUSION IN $d=1$ PATTERN FORMATION



$$\Gamma = A e^{iq_M x} + c.c.$$

$$A = R e^{i\psi} e^{iqx}$$

PHASE MODE:

$$x \rightarrow x_0 + x \Rightarrow \psi \rightarrow \psi_0 + \psi$$

PHASE EQUATION:

$$\partial_t \psi = D(q) \partial_x^2 \psi + \xi(x, t)$$

ECKHAUS STABLE:

$$D(q) > 0$$

PHASE FLUCTUATIONS:

$$\langle \psi_k^2 \rangle \approx k^{-2}$$

$$d = 2 \Rightarrow \int d\mathbf{k} \langle \psi_k^2 \rangle \approx \ln L$$

Symmetry restoring by noise:

NO LONG RANGE ORDER FOR $d < d_c = 2$

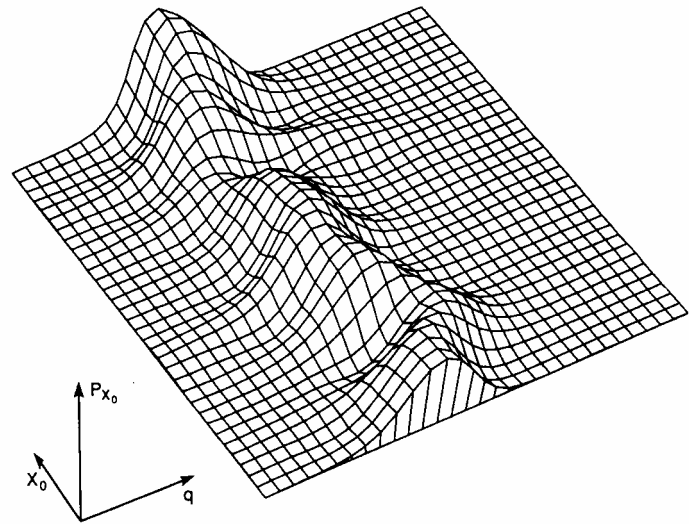
STOCHASTIC $d=1$ SWIFT-HOHENBERG EQUATION

$$\partial_t \Gamma(x,t) = \left[\mu - (q_M + \partial_x^2)^2 \right] \Gamma - \Gamma^3 + \sqrt{\varepsilon} \xi(x,t)$$

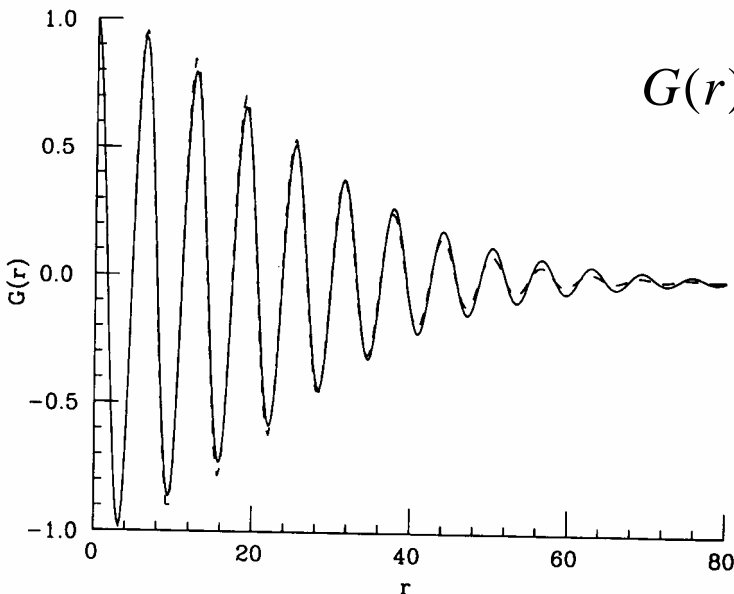
LOCAL POWER SPECTRUM:

$$P_{x_0} \approx \left(\Gamma_q \Big|_{x_0}^2 \right)$$

- **LOCAL WAVE NUMBER**
- **BROAD SPECTRUM**



CORRELATION FUNCTION:



$$G(r) = G(0) e^{-\left(\frac{r}{r_0}\right)^2} \cos(q_M r)$$

Correlation length $r_0 \sim L/50$

NO LONG RANGE ORDER



FLUCTUATIONS AND STRIPE PATTERNS ABOVE THRESHOLD

$$O(\mathbf{r}, t) = A_+ e^{iq_M x} + A_- e^{-iq_M x}; \quad A_+ = R e^{i(\phi+\psi)}, \quad A_- = R e^{i(\phi-\psi)}$$

$$O(\mathbf{r}, t) = R e^{i\phi} \cos(q_M x + \psi)$$

SWIFT-HOHENBERG EQ.

$$\partial_t \Gamma(\mathbf{r}, t) = \left[\mu - (q_M + \nabla^2)^2 \right] \Gamma - \Gamma^3 + \sqrt{\epsilon} \xi(\mathbf{r}, t)$$

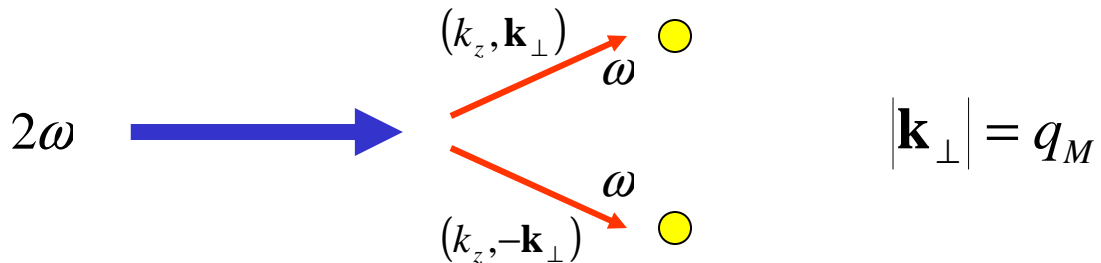
$O(\mathbf{r}, t)$ real: $A_+ = A_-^*$, $\phi = 0$, ψ arbitrary

Noise effects: $\left\{ \begin{array}{l} I_+ = I_- \text{ for any noise event } (I_{\pm} = |A_{\pm}|^2) \\ \psi \text{ diffuses} \end{array} \right.$

Degenerate OPO:

$$\begin{aligned} \partial_t A_0 &= \gamma_0 \left[-(1 + i\Delta_0) A_0 + E_0 + ia_0 \nabla^2 A_0 + 2iK_0 A_1^2 \right] + \sqrt{\epsilon_0} \xi_0(\mathbf{r}, t) \\ \partial_t A_1 &= \gamma_1 \left[-(1 + i\Delta_1) A_1 + ia_1 \nabla^2 A_1 + iK_0 A_1^* A_0 \right] + \sqrt{\epsilon_1} \xi_1(\mathbf{r}, t) \end{aligned}$$

$O(\mathbf{r}, t) \Rightarrow A_1$ complex, $q_M(\Delta_0)$, $\phi(\Delta_1)$ fixed, ψ arbitrary



Noise effects: $\left\{ \begin{array}{l} I_+ - I_- \text{ small } (I_{\pm} = |A_{\pm}|^2) \\ \psi \text{ large fluctuations} \end{array} \right.$ **COMPLEMENTARITY**



NOISY PRECURSORS and QUANTUM IMAGES

SWIFT-HOHENBERG EQ.

$$\partial_t \Gamma(\mathbf{r}, t) = \left[\mu - (q_M + \nabla^2)^2 \right] \Gamma - \Gamma^3 + \sqrt{\epsilon} \xi(\mathbf{r}, t)$$

Linear analysis below threshold:

$$\partial_t \Gamma_{\mathbf{k}} = \omega(|\mathbf{k}|^2) \Gamma_{\mathbf{k}} + \sqrt{\epsilon} \xi_{\mathbf{k}}, \quad \Gamma_{\mathbf{k}} = \Gamma_{-\mathbf{k}}^*, \quad \omega(|\mathbf{k}|^2) = (\mu - (q_M - |\mathbf{k}|^2)^2) < 0$$

Power Spectrum ("Far field"):

$$I_{\mathbf{k}} = I_{-\mathbf{k}} = |\Gamma_{\mathbf{k}}|^2, \quad \langle I_{\mathbf{k}} \rangle = -\frac{\epsilon}{\omega(|\mathbf{k}|^2)}$$

$$\langle \Delta I_{\mathbf{k}}(t) \Delta I_{\mathbf{k}'}(t) \rangle = (\delta_{\mathbf{k}, \mathbf{k}'} + \delta_{\mathbf{k}, -\mathbf{k}'}) \frac{\epsilon^2}{\omega^2(|\mathbf{k}|^2)} I_{\mathbf{k}}$$

$\left\{ \begin{array}{l} \text{Ring of maximum power: } |\mathbf{k}|^2 = q_M \\ \mathbf{k} \text{ correlated with } \pm \mathbf{k} \text{ (} \Gamma \text{ real)} \end{array} \right.$

Degenerate OPO:

$$\partial_t A_0 = \gamma_0 \left[-(1 + i\Delta_0) A_0 + E_0 + ia_0 \nabla^2 A_0 + 2iK_0 A_1^2 \right] + \sqrt{\epsilon_0} \xi_0(\mathbf{r}, t)$$

$$\partial_t A_1 = \gamma_1 \left[-(1 + i\Delta_1) A_1 + ia_1 \nabla^2 A_1 + iK_0 A_1^* A_0 \right] + \sqrt{\epsilon_1} \xi_1(\mathbf{r}, t)$$

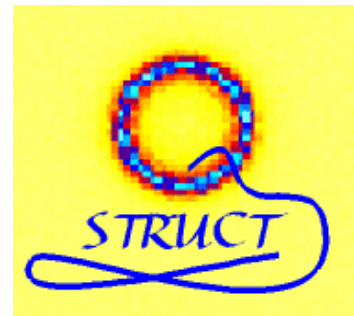
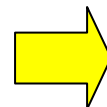
Linear analysis below threshold:

$$\partial_t A_{1,\mathbf{k}} = \nu(|\mathbf{k}|^2) A_{1,\mathbf{k}} + iK_0 A_{1,-\mathbf{k}}^* A_0^s + \sqrt{\epsilon_1} \xi_{\mathbf{k}}, \quad \nu(|\mathbf{k}|^2) = \gamma_1 \left[-(1 + i\Delta_1) - ia_1 |\mathbf{k}|^2 \right]$$

Power Spectrum ("Far field"):

$$I_{\mathbf{k}} = |A_{1,\mathbf{k}}|^2, \quad I_{\mathbf{k}} \neq I_{-\mathbf{k}} \text{ but } \mathbf{k} \text{ and } -\mathbf{k} \text{ linearly coupled}$$

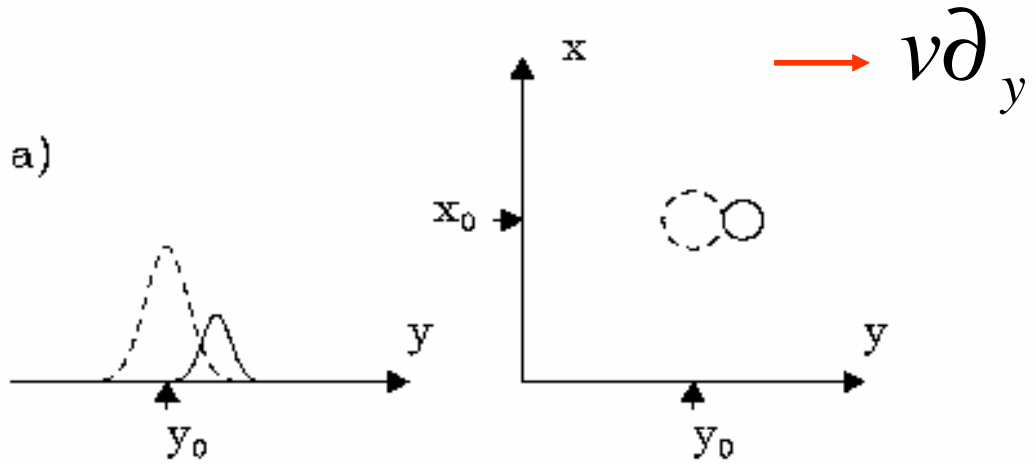
not a simple pattern
Complex structure



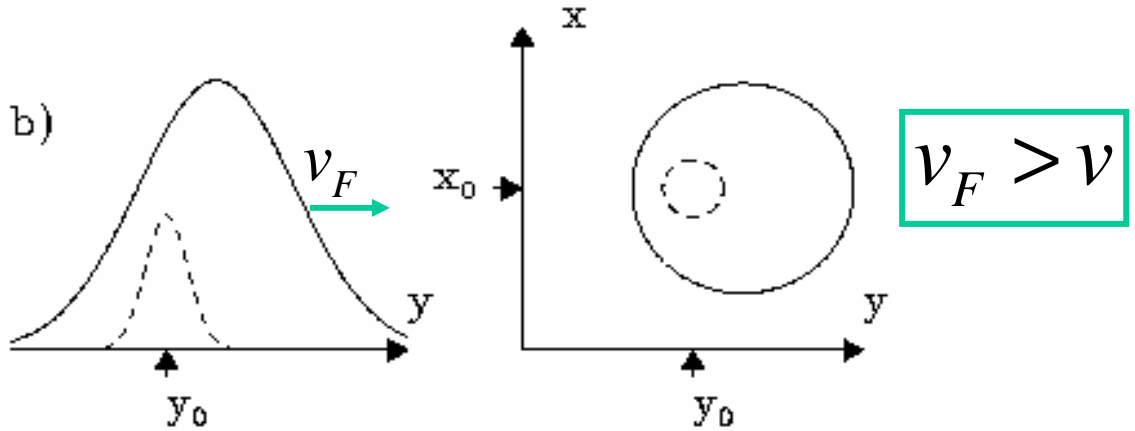
Beware: Pattern selection is a nonlinear mechanism !!

CONVECTIVE INSTABILITY

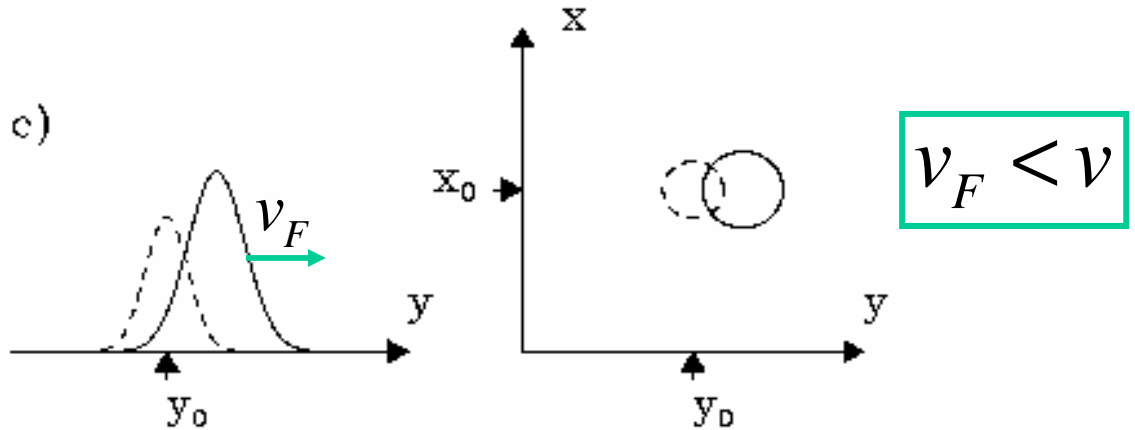
Absolutely Stable



Absolutely Unstable



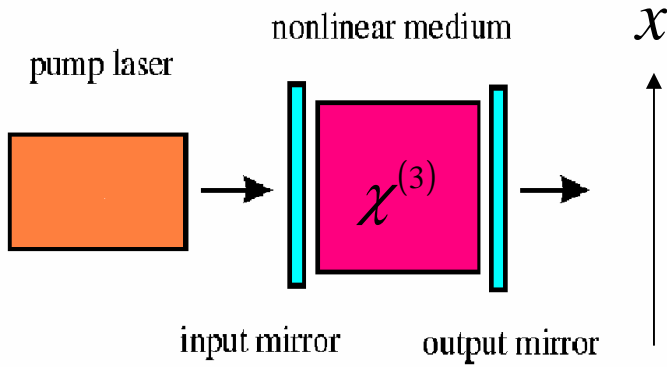
Convectively Unstable



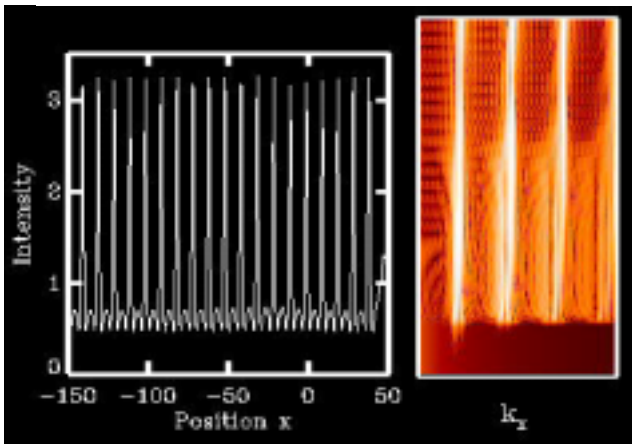
Absolute Instability Threshold: $v = v_F$

**Convectively Unstable Regime:
Noise Sustained Structure**

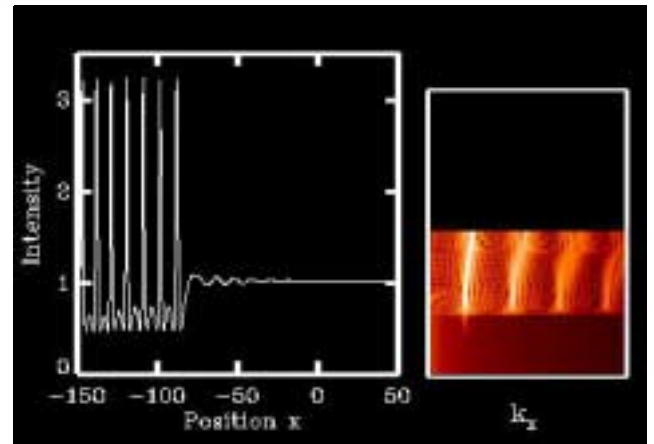
Noise Sustained Structure in a Kerr Resonator



∂_x : Tilted Pump

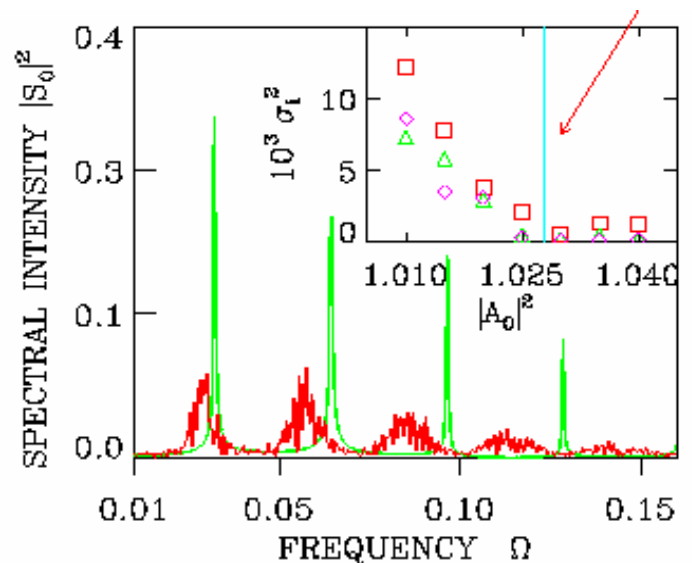


Absolutely Unstable



Convectively Unstable

Spectral Narrowing identifies transition from a NSS to a Deterministic Pattern





CONCEPTS IN NOISE SUSTAINED STRUCTURES

PHENOMENON OF NOISE AMPLIFICATION, while quantum images are weakly damped critical fluctuations

NOISE NEEDED AT ALL TIMES: A laser requires amplification of spontaneous emission, but when the laser is lasing noise is no longer required to maintain the oscillation

PRECURSOR OF AN ABSOLUTELY UNSTABLE REGIME:
An optical amplifier is in a convective regime, but there is no regime of absolute instability

Examples

Fabry-Perot Cavities filled with
a cubic (Kerr) nonlinear medium
Transverse drift due to tilted pump

Ring optical parametric oscillators
(quadratic nonlinearity)
Transverse walk-off due to birefringence

One-dimensional
(roll pattern)

Two-dimensional
(hexagonal pattern)

Type I
(one polarization)

Type II
(two polarizations)

Santagiustina et al., Phys
Rev. Lett. 79, 3633 (97)

Santagiustina et al., Quant.
Sem. Opt., to appear (99)

Santagiustina et al., Phys
Rev. E 58, 3843 (97)



Type II Optical Parametric Oscillator

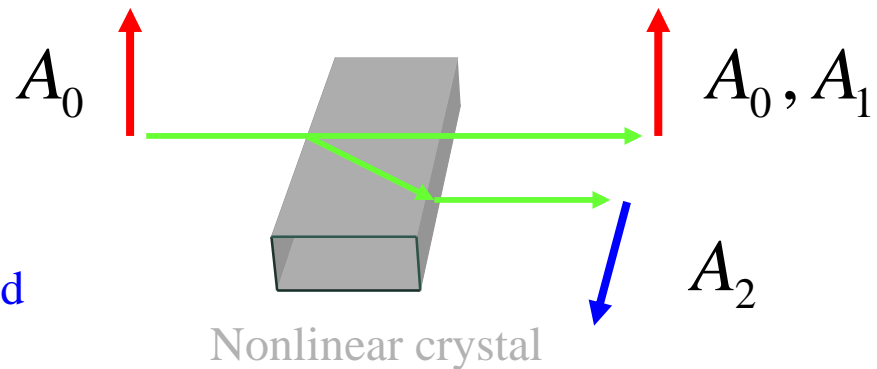
$$\partial_t A_0 = \gamma_0 \left[-(1 + i\Delta_0) A_0 + E_0 + ia_0 \nabla^2 A_0 + 2iK_0 A_1 A_2 \right] + \sqrt{\epsilon_0} \xi_0(\mathbf{r}, t)$$

$$\partial_t A_1 = \gamma_1 \left[-(1 + i\Delta_1) A_1 + ia_1 \nabla^2 A_1 + iK_0 A_2^* A_0 \right] + \sqrt{\epsilon_1} \xi_1(\mathbf{r}, t)$$

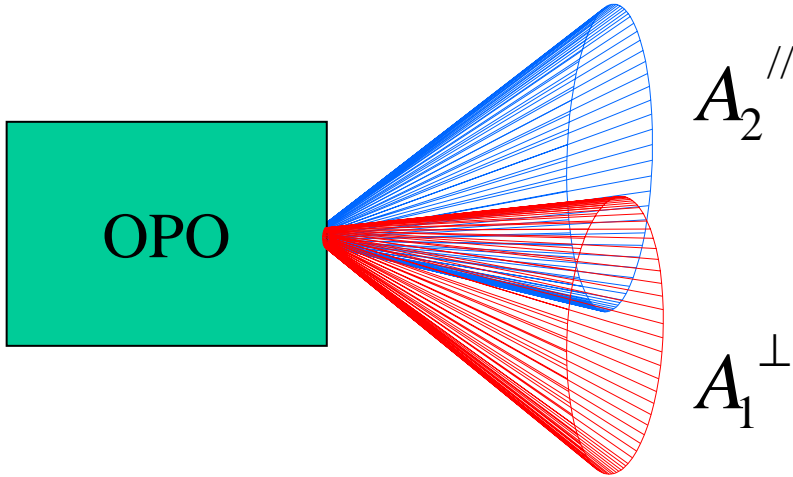
$$\partial_t A_2 = \gamma_2 \left[-(1 + i\Delta_2) A_2 + ia_2 \nabla^2 A_2 + iK_0 A_1^* A_0 + \rho_2 \partial_y A_2 \right] + \sqrt{\epsilon_2} \xi_2(\mathbf{r}, t)$$

Walk-off

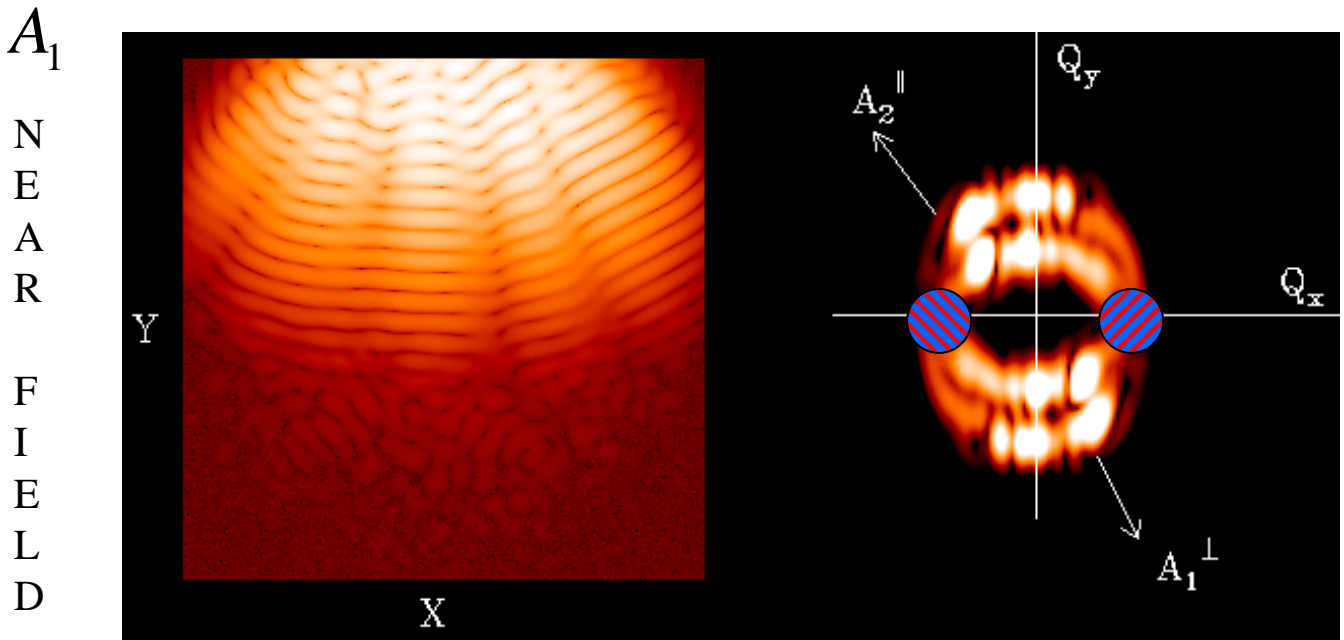
- A_0 Pump field, Ordinary Polarized
- A_1 Signal field, Ordinary Polarized
- A_2 Idler field, Extraordinary Polarized

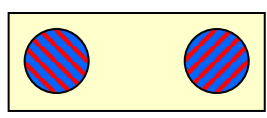


Type II OPO: CONVECTIVE REGIME



**Noise Amplification
in Optical Cavity**



- State**
- 
- 1) Nonlinear Q. Correlations $\mathbf{k}_\perp, -\mathbf{k}_\perp$**
 - 2) Polarization Entanglement in a macroscopic Noise Sustained Structure**